

Spin-Hall Conductivity in Electron-Phonon Coupled Systems

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We derive the ac spin-Hall conductivity $\sigma_{\text{SH}}(\omega)$ of two-dimensional spin-orbit coupled systems interacting with dispersionless phonons of frequency ω_0 . For the linear Rashba model we show that the electron-phonon contribution to the spin-vertex corrections breaks the universality of $\sigma_{\text{SH}}(\omega)$ at low-frequencies and provides a non-trivial renormalization of the interband resonance. On the contrary, in a generalized Rashba model for which the spin-vertex contributions are absent, the coupling to the phonons enters only through the self-energy, leaving the low frequency behavior of $\sigma_{\text{SH}}(\omega)$ unaffected by the electron-phonon interaction.

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The recent prediction of intrinsic spin currents generated by applied electric fields in semiconductors with spin-orbit (SO) interaction [1, 2] has attracted intensive research on the subject [3, 4] encouraged also by potential applications in spintronic-based devices. In such systems, the spin-Hall conductivity $\sigma_{\text{SH}} = J_y^{S_z}/E_x$, where $J_y^{S_z}$ is a spin S_z polarized current in the y -direction and E_x is the electric field directed along x , arises from the SO dependent band structure which, for clean systems, leads for example to $\sigma_{\text{SH}} = -e/8\pi$ for a two-dimensional (2D) electron system with Rashba SO coupling [2] or to $\sigma_{\text{SH}} = -3e/8\pi$ for a 2D hole semiconductor [5, 6].

Of special interest for both applied and fundamental research is the role played by scattering events which have been shown to modify in an essential way the clean limit results. The most drastic effects are found in the 2D linear Rashba model, where σ_{SH} reduces to zero for arbitrarily weak impurity scattering [7, 8, 9, 10, 11], while the universal value $\sigma_{\text{SH}}(\omega) = -e/8\pi$ is recovered for finite values of the ac field frequency ω in the range $\tau^{-1} < \omega < \Delta$ [8, 10], where τ^{-1} is the impurity scattering rate and Δ is the spin-orbit energy splitting. On the contrary, in 2D hole systems with weak (short ranged) impurity scattering, $\sigma_{\text{SH}}(\omega)$ remains equal to $-3e/8\pi$ for $0 \leq \omega < \Delta$ [5, 6, 13], while it becomes dependent on the impurity potential if this has long range character [14, 15].

So far, the study of scattering effects on the spin-Hall conductivity has been restricted to the case in which the source of scattering is the coupling of the charge carriers to some elastic impurity potential. This leaves aside the contributions from inelastic scattering such those provided by the electron-phonon (el-ph) interaction which, in the materials of interest for the spin-Hall effect, ranges from the weak-coupling limit in GaAs [16] to the strong-coupling regime in Bi(100) [17].

Because of its dynamic and inelastic character, the el-ph interaction may affect the spin-Hall response in a way

drastically different from static elastic impurity scattering, questioning the general validity of the commonly accepted forms of $\sigma_{\text{SH}}(\omega)$ summarized above. Furthermore, the issue of the vertex corrections, which are responsible for the vanishing of $\sigma_{\text{SH}}(\omega = 0)$ in the impure 2D linear Rashba model [7, 9, 10, 11, 12], acquires a new importance, since these should be altered by the el-ph interaction.

In this letter we report on our results on the spin-Hall conductivity $\sigma_{\text{SH}}(\omega)$ for 2D systems with SO interaction coupled with dispersionless phonons of frequency ω_0 . For a linear Rashba model we show that, in the frequency range $\tau^{-1} < \omega \ll \Delta$ (with $\Delta < \omega_0$) where the universal value $-e/8\pi$ has been predicted, the el-ph contribution to the vertex corrections reduces $\sigma_{\text{SH}}(\omega)$ to the nonuniversal value $-e/[8\pi(1 + \lambda/2)]$, where λ is the el-ph coupling constant. Furthermore, we find that the el-ph spin-vertex contributions renormalize also the interband transitions and provide a further reduction of $\sigma_{\text{SH}}(\omega)$ for $\omega > \omega_0$. On the contrary, in a 2D generalized Rashba model, for which the spin-vertex contributions are absent, the el-ph interaction provides only a trivial self-energy correction to the interband transition, leaving the low frequency part of $\sigma_{\text{SH}}(\omega)$ basically unaltered.

We consider the el-ph interaction as given by the Holstein hamiltonian generalized to include SO coupling:

$$H = \sum_{\mathbf{k}, \alpha} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} + \sum_{\mathbf{k}, \alpha, \beta} \Omega_{\mathbf{k}} \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\beta} + \omega_0 \sum_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + g \sum_{\mathbf{q}, \mathbf{k}, \alpha} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}\alpha} (a_{\mathbf{q}} + a_{-\mathbf{q}}^\dagger), \quad (1)$$

where $c_{\mathbf{k}\alpha}^\dagger$ and $a_{\mathbf{q}}^\dagger$ ($c_{\mathbf{k}\alpha}$ and $a_{\mathbf{q}}$) are the creation (annihilation) operators for an electron with momentum $\mathbf{k} = (k_x, k_y)$ and spin index $\alpha = \uparrow, \downarrow$, and for a phonon with wavenumber \mathbf{q} . $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ is the electron dispersion, ω_0 is the phonon frequency and g is a momentum independent el-ph interaction (Holstein model). The use

of a Holstein coupling permits to focus solely on the retardation and inelastic effects of phonons, disentangling the study from possible momentum dependences of the el-ph interaction. Furthermore, the Holstein coupling is partially justified, for example, by the results on surface states [17] and by the reduced momentum dependence, compared to 3D electron gases, of 2D electrons coupled to bulk polar optical phonons [16]. In the following, we shall also include the coupling to a short-ranged impurity potential $V(\mathbf{r}) = V_{\text{imp}} \sum_i \delta(\mathbf{r} - \mathbf{R}_i)$, where \mathbf{R}_i are the random positions of the impurity scatterers.

Let us start by considering a linear Rashba model, for which the SO vector potential is $\mathbf{\Omega}_{\mathbf{k}} = \gamma k(-\sin \phi, \cos \phi)$, where γ is the SO coupling and ϕ is the polar angle. The electron Green's function of the interacting system is

$$G(\mathbf{k}, i\omega_n) = \frac{1}{2} \sum_{s=\pm 1} [1 + s\hat{\mathbf{\Omega}}_{\mathbf{k}} \cdot \boldsymbol{\sigma}] G_s(k, i\omega_n), \quad (2)$$

where $\hat{\mathbf{\Omega}}_{\mathbf{k}} = (-\cos \phi, \sin \phi)$ and $G_s(k, i\omega_n) = [i\omega_n - E_k^s + \mu - \Sigma(i\omega_n)]^{-1}$ is the Green's function in the helicity basis with dispersion $E_k^s = \hbar^2(k + sk_0)^2/2m$, $k_0 = m\gamma/\hbar^2$ is the SO wavenumber, μ is the chemical potential and $\omega_n = (2n+1)\pi T$ is the fermionic Matsubara frequency at temperature T . Due to the momentum independence of g and V_{imp} , the self-energy $\Sigma(i\omega_n)$ is independent of \mathbf{k} and reduces to

$$\Sigma(i\omega_n) = T \sum_{n'} \frac{W(i\omega_n - i\omega_{n'})}{2N_0} \sum_{s=\pm} \int \frac{dk}{2\pi} k G_s(k, i\omega_{n'}), \quad (3)$$

where $N_0 = m/2\pi\hbar^2$ is the density of states per spin direction and

$$W(i\omega_n - i\omega_{n'}) = \frac{\delta_{n,n'}}{2\pi\tau T} - \lambda \frac{\omega_0^2}{(i\omega_n - i\omega_{n'}) - \omega_0^2}, \quad (4)$$

where $\tau^{-1} = 2\pi n_i V_{\text{imp}}^2 N_0$ is the impurity scattering rate and $\lambda = 2g^2 N_0/\omega_0$ is the el-ph coupling. In writing Eqs.(3,4), we have employed the self-consistent Born approximation for both impurity and el-ph scatterings.

The equations defining the spin-Hall conductivity are obtained from the Kubo formula applied to the el-ph problem. Hence, the spin-current-charge-current correlation function is

$$K(i\nu_m) = i \frac{e\hbar^2\gamma}{4m} T \sum_n \Gamma(i\omega_l, i\omega_n) B_1(i\omega_l, i\omega_n), \quad (5)$$

where $\nu_m = 2m\pi T$ is a bosonic Matsubara frequency, $\omega_l = \omega_n + \nu_m$ and

$$B_1(i\omega_l, i\omega_n) = \int \frac{dk}{2\pi} k^2 \sum_s G_{-s}(k, i\omega_l) G_s(k, i\omega_n). \quad (6)$$

The vertex function Γ appearing in Eq.(5) satisfies the

following self-consistent equation

$$\Gamma(i\omega_l, i\omega_n) = 1 + T \sum_{n'} \frac{W(i\omega_{n'} - i\omega_n)}{4N_0 k_0} [B_2(i\omega_{l'}, i\omega_{n'}) + k_0 B_3(i\omega_{l'}, i\omega_{n'}) \Gamma(i\omega_{l'}, i\omega_{n'})], \quad (7)$$

where $\omega_{l'} = \omega_{n'} + \nu_m$ and

$$B_2(i\omega_l, i\omega_n) = \int \frac{dk}{2\pi} k^2 \sum_s G_s(k, i\omega_l) G_s(k, i\omega_n), \quad (8)$$

$$B_3(i\omega_l, i\omega_n) = \int \frac{dk}{2\pi} k \sum_{s,s'} G_s(k, i\omega_l) G_{s'}(k, i\omega_n). \quad (9)$$

All integrations over the momenta k appearing in the above equations can be performed analytically, while the self-consistent equations (3) and (7) are solved numerically by iteration in the Matsubara frequency space. Finally, the (complex) spin-Hall conductivity

$$\sigma_{\text{SH}}(\omega) = i \frac{K^R(\omega)}{\omega} \quad (10)$$

is obtained from the retarded function $K^R(\omega) \equiv K(\omega + i\delta)$ extracted from $K(i\nu_m)$ [Eq.(5)] by applying the Padé method of numerical analytical continuation. Although our numerical calculations can be applied to arbitrary values of Δ/E_F , where E_F is the Fermi energy and $\Delta = 2\gamma k_F$ is the SO splitting, the following discussion will be restricted to the weak SO coupling limit $\Delta/E_F \ll 1$, common to many materials, for which some analytical results can be obtained. In our calculations we have used $T = 0.01\omega_0$ (or $T = 0.001\omega_0$ for the case shown in Fig. 2), which is representative of the zero temperature case.

We start our analysis by considering first the case $\omega_0 > \Delta$ for which, as discussed below, the el-ph effects enter mainly through the real parts of the self-energy and of the vertex function. In Fig. 1 we show the real and imaginary parts of the spin-Hall conductivity for $\omega_0 = 1.5\Delta$ and $\lambda = 0, 0.5, 1.0$, and for weak impurity scattering $1/\tau\Delta = 0.05$. In the absence of el-ph interaction ($\lambda = 0$), we recover the known results [8, 10] characterized by the strong interband transitions at $\omega = \Delta$ and by the vanishing of $\sigma_{\text{SH}}(\omega)$ as $\omega \rightarrow 0$. Furthermore, in the intermediate-frequency region $1/\tau < \omega \ll \Delta$, $\text{Re } \sigma_{\text{SH}}(\omega)$ is almost ω -independent and matches the universal value $-e/8\pi$. This is better displayed in Fig. 2(a) where the low-frequency behavior is plotted for $1/\tau\Delta = 0.005$. Upon enhancing λ , two new features emerge. Namely, the frequency of the interband transitions get shifted at a lower (λ dependent) value and, as also shown in Fig. 2(a), the intermediate-frequency real spin-Hall conductivity deviates from $-e/8\pi$, indicating that universality breaks down when $\lambda \neq 0$. The origin of these features can be understood from the analysis of Eqs. (5) and (7). In fact, at zero temperature and for $\Delta/E_F \ll 1$, the retarded function $K^R(\omega)$ reduces to [18]

$$K^R(\omega) = \frac{e\hbar^2\gamma}{4m} \int_{-\omega}^0 \frac{d\epsilon}{2\pi} \Gamma(\epsilon_+ + \omega, \epsilon_-) B_1(\epsilon_+ + \omega, \epsilon_-), \quad (11)$$

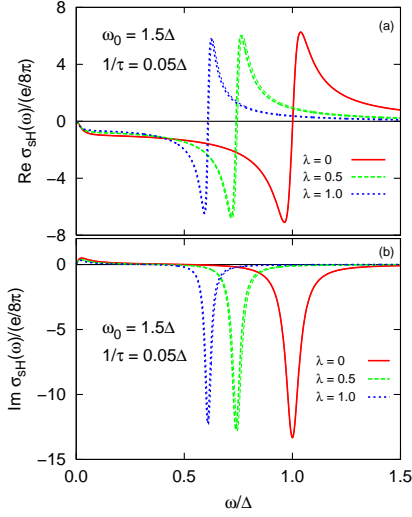


FIG. 1: (color online) Real (a) and imaginary (b) parts of the spin-Hall conductivity for $\omega_0 > \Delta$ obtained from the numerical analytical continuation of Eqs.(5)-(10) (thick lines). The analytical formula (12) is plotted with thin lines and is almost indistinguishable from the numerical results. The peaks of $\text{Im } \sigma_{\text{SH}}(\omega)$ are centered at $\omega/\Delta = \sqrt{1 + \lambda/2}/(1 + \lambda)$.

where $\epsilon_{\pm} = \epsilon \pm i\delta$. For $\omega < \omega_0$, the integration appearing in (11) restricts the $\epsilon + \omega$ and ϵ variables to $|\epsilon + \omega| < \omega_0$ and $|\epsilon| < \omega_0$, for which the self-energy on the real axis can be well approximated by $\Sigma(x_{\pm}) = -\lambda \text{Re}(x_{\pm}) \mp i/2\tau$, where $x_- = \epsilon_-$ and $x_+ = \epsilon_+ + \omega$. In this way, the quite lengthy integral equation for $\Gamma(\epsilon_+ + \omega, \epsilon_-)$, which can be derived from Eq.(7) by following the method of analytic continuation described in Ref.[18], reduces to a simple ω -dependent algebraic equation. Its solution for $\omega/\Delta \ll 1$ is $\Gamma(\omega) \simeq \omega/[(1 + \lambda/2)\omega + i/2\tau]$ and, since $B_1(\omega)$ is a constant for $\omega/\Delta \ll 1$, the low-frequency spin-Hall conductivity becomes $\sigma_{\text{SH}}(\omega) \simeq -(e/8\pi)\Gamma(\omega)$. We recover therefore the vanishing of $\sigma_{\text{SH}}(\omega)$ for $\omega \rightarrow 0$ while, contrary to the $\lambda = 0$ case, we find that $\sigma_{\text{SH}}(\omega)$ is approximately equal to the nonuniversal constant $-e/[8\pi(1 + \lambda/2)]$ for $1/\tau < \omega \ll \Delta$. The breakdown of universality at intermediate frequency reported in Figs. 1(a) and 2(a) stems therefore from the el-ph contribution to the spin-vertex correction which, from Eqs.(7-9), governs the intraband contributions to $\sigma_{\text{SH}}(\omega)$. A more refined calculation which takes into account also the interband transitions leads to:

$$\sigma_{\text{SH}}(\omega) = -\frac{e}{8\pi} \frac{\omega}{(1 + \frac{\lambda}{2})\omega + \frac{i}{2\tau} - [(1 + \lambda)\frac{\omega}{\Delta} + \frac{i}{\tau\Delta}]^2 \omega}, \quad (12)$$

which is valid for $\omega < \omega_0$ and arbitrary ω/Δ (for $\Delta/E_F \ll 1$). For $\lambda = 0$, Eq.(12) is identical to the formula already published in Refs.[8, 10]. Instead, for $\lambda > 0$ we recover the intermediate frequency nonuniversal behavior discussed above together with an el-ph renormalization effect to the interband transitions, which

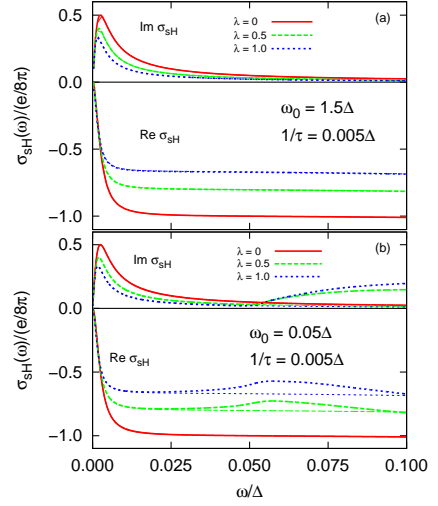


FIG. 2: (color online) Low frequency behavior of $\sigma_{\text{SH}}(\omega)$ for $\omega_0 > \Delta$ (a) and for $\omega_0 \ll \Delta$ (b). The thick lines are the numerical results, while the thin lines are Eq.(12). In panel (a) they are barely distinguishable.

now occur at a frequency $\omega = \Delta^*$, where

$$\Delta^* = \frac{\sqrt{1 + \lambda/2}}{1 + \lambda} \Delta \quad (13)$$

for $1/\tau\Delta \ll 1$. When compared with the numerical results of Figs. 1 and 2(a), equation (12) is in excellent agreement for all frequencies lower than ω_0 . As a matter of fact, Eq.(12) is in very good agreement with the numerical results also for $\omega > \omega_0$ as long as $\omega_0 > \Delta$ while, for $\omega_0 < \Delta$, the ω -dependence of $\sigma_{\text{SH}}(\omega)$ starts to be affected by the imaginary contributions of the el-ph self-energy and of the vertex function. These effects are visible in Fig. 2(b), where we compare the numerical results for $\omega_0 = 0.05\Delta$ (thick lines) with Eq.(12) (thin lines). The deviation of $\text{Re } \sigma_{\text{SH}}(\omega)$ from $-e/[8\pi(1 + \lambda/2)]$ for $\omega \gtrsim \omega_0$ stems from intraband transitions mediated by the phonons which, in analogy to the low temperature optical conductivity of the Holstein el-ph model [19, 20], ensure conservation of energy and momentum. At higher frequencies, the real part of the el-ph self-energy goes to zero as $\lambda\omega_0^2/\omega$ for large ω/ω_0 , and the interband transitions occur at the unrenormalized frequency $\omega \approx \Delta$.

Having established that the nonuniversality of $\sigma_{\text{SH}}(\omega)$ at intermediate frequencies and the non-trivial renormalization (13) have their origin in the el-ph contributions to the spin-vertex correction, we now turn to evaluate the el-ph effects when the spin-vertex corrections are absent. To investigate this point we have considered a 2D generalized Rashba model where the SO interaction is of the form $\mathbf{\Omega}_{\mathbf{k}} = \gamma k^N (-\sin N\phi, \cos N\phi)$ [15]. For $N = 1$, the linear Rashba model discussed above is recovered, while for $N = 3$ this model describes a 2D hole gas subjected to an asymmetric confining potential. Because of the

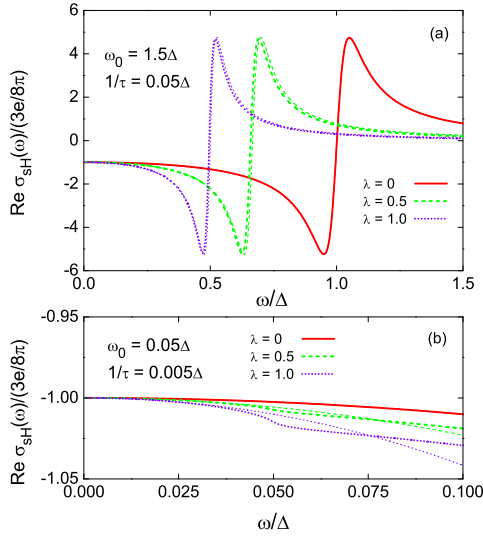


FIG. 3: (color online) (a): Real part of the spin-Hall conductivity for a generalized Rashba model with $N = 3$. Thick lines are the numerical results while thin lines are Eq.(14). (b): low-frequency region for $\omega_0 \ll \Delta$.

angular dependence of $\mathbf{\Omega}_k$ for $N \neq 1$, the vertex corrections are absent [6], and the correlation function $K(i\nu_m)$ is simply given by Eq.(5) with $\Gamma(i\omega_l, i\omega_n) = 1$ and with the prefactor multiplied by N . Furthermore, the function $B_1(i\omega_l, i\omega_n)$ is as given in Eq.(6), with dkk^2 replaced by dkk^{1+N} and with dispersion $E_k^s = \hbar^2 k^2/2m + s\gamma k^3$. Contrary to the linear Rashba model, now all el-ph effects arise solely from the self-energies contained in the interband bubble term B_1 . Hence, in the weak SO limit $\Delta/E_F \ll 1$, where now $\Delta = 2\gamma k_F^3$, and by using the same approximation scheme as above, for $\Delta, \omega < \omega_0$ the spin-Hall conductivity is easily found to be given by:

$$\sigma_{\text{SH}}(\omega) = -\frac{eN}{8\pi} \frac{\Delta^2}{\Delta^2 - [(1+\lambda)\omega + i/\tau]^2}. \quad (14)$$

Contrary to Eq.(12), the above expression predicts a low-frequency behavior unaffected by the el-ph interaction. Namely: $\sigma_{\text{SH}}(\omega) = -eN/8\pi$ for $\omega \ll \Delta$. Furthermore, the interband transition frequency is renormalized only by the el-ph self-energy (mass enhancement) factor $1+\lambda$: $\Delta^* = \Delta/(1+\lambda)$, in contrast with Eq.(13) where the el-ph contribution to the spin-vertex corrections contributes with a factor $\sqrt{1+\lambda/2}$. This behavior is confirmed by our numerical results for $N = 3$ reported in Fig. 3(a) (thick lines), which fully agree with Eq. (14) (thin lines). Furthermore, as shown in Fig. 3(b) for $\omega_0 = 0.05\Delta$ and $1/\tau\Delta = 0.005$, for $\omega \gtrsim \omega_0$ we find a weak deviation from Eq.(14) due solely to the imaginary part of the self-energy, in contrast to Fig. 2(b) where the spin-vertex corrections have a much stronger effect.

Before concluding, it is worth discussing how our results can be obtained experimentally. In particular for

the 2D linear Rashba model we can make use of the equivalence between $\sigma_{\text{SH}}(\omega)$ and the longitudinal in-plane spin susceptibility $\chi_{\parallel}(\omega)$ [11], and directly relate the poles of $\sigma_{\text{SH}}(\omega)$ with the time evolution of the spin polarization $S_y(t)$ [10], which can be measured by various techniques [21]. We find thus from Eq.(12) that for $\tau\Delta \gg 1$, $S_y(t)$ is a function oscillating with frequency Δ^* , Eq.(13), damped by an exponential decay with rate $1/\tau_s = 1/[2\tau(1+\lambda/2)]$. On the contrary, the decay rate in the $\tau\Delta \ll 1$ limit is independent of the el-ph interaction at $T = 0$, and reduces to the Dyakonov-Perel value $1/\tau_s = \Delta^2\tau/2$ [21].

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